# Weight Functions for Two-Dimensional Arrays Using Regularized Deconvolution

James M. Alsup
AETC Incorporated
8910 University Center Lane, Suite 900
San Diego, California 92122
alsup@sd.aetc.com

Harper J. Whitehouse
Linear Measurements, Inc.
4174 Sorrento Valley Blvd.
San Diego, CA 92121
whitehouse@linearmeasurements.com

## Abstract

*A method for computing sidelobe-reducing weight matrices for planar non-uniform array geometries is described. These matrices are applied in the beam-space domain, are a function of frequency, and can be precomputed and stored for later use if the array geometry remains relatively fixed (they are signal-data independent). The method is based on the use of constrained, regularized deconvolution techniques to robustly reduce sidelobes and retain or improve the intrinsic angular resolution of the array.*

## Introduction

The sidelobe-reducing weights are being developed in the context of sonar systems, active or passive, in which an acoustic array is used to detect the presence of undersea targets against a background of acoustic noise and reverberation. The signals from the acoustic array elements might be processed in the manner shown in Figure 1. Within this process, the signals are beamformed, and within the beamforming, the sidelobe-reducing weights are applied. Unlike the more traditional vector weights used with uniform line arrays, these matrix weights are applied in the beamspace domain, with separate matrices for each frequency component. Like the traditional vector weights, these matrix weights can be pre-computed and stored for later use if the array itself does not change shape appreciably during use (i.e., the weights are not data-dependent).

In figure 1a, **x** represents a data vector whose length M is the number of elements in the array, and **S** represents the matrix (size NxM) of steering phases used represent plane-wave arrivals from N horizontal directions at the array. The matrix-vector product ( **W** * **S'** * **x** ) refers to the operation of applying an NxN weight matrix, **W**, to the conventionally-formed beams, (**S'** * **x**). In this paper, we are using Matlab notation, where the "prime" superscript refers to conjugate-transpose. Finally, to preserve the signal phase, the magnitude of (**W** * **S'** * **x**) is element-multiplied ( .* ) by the original phase of the vector ( **S'** * **x̲** ) .

## The Arrays

Several planar arrays have been considered: a 40-element annular planar array is used as the primary example in this paper (Figure 2a); a 40-element filled planar array is used as the secondary example (Figure 2b). These array designs are notionally suitable for the purpose of detecting sonar targets if deployed as receivers in a drifting or bottomed sensor system.

## The Deconvolutional Beamformer (DBF) Algorithm

### a) Steering Matrices

The conventional beamformer steering matrix for single-frequency plane waves with zero elevation can be specified as follows:

**S**(f) = exp( (j*2* ð / ë) *( **x***cos(**è**) + **y***sin(**è**)), where

**è** = vector of azimuth steer directions, 1xN
**x** and **y** = vectors of element positions, Mx1 (each)
ë = wavelength = c / f,

f = analysis frequency component
c = sound velocity

The steering matrix **S** has size NxM, in which each row represents a steering vector for one of the N azimuths being steered.

### b) Conventional Beam Patterns

The point-spread matrix of the array is given by:

$$\mathbf{A} = (\mathbf{S}') * \mathbf{S} .$$

The size of **A** is NxN. Each row of this matrix also represents a CBF beampattern vector for one of the N steering directions being implemented. The magnitude |**A**| of the point-spread matrix for N=45 is shown for the annular array in Figure 3a.

### c) Regularized Deconvolution

Conceptually, the first step in computing **W** is to deconvolve **A**. In the absence of element location uncertainty, the traditional Moore-Penrose pseudo-inverse of **A**, denoted by $\mathbf{A}^+$, is used. However, in order to stabilize (regularize) this computation, a more flexible computational method must be used. We designate the result $\mathbf{C} = \mathbf{A}_k^+$, and compute **C** using the regularization method of Per Christian Hansen [ref 2]. Note that **A**, **C**, and **W** are all functions of frequency. The following steps are followed:

a) compute [**u**,**s**,**v**] = svd( **A** ).

b) compute **si**, the regularized diagonal inverse of the diagonal matrix **s**, by inverting the first k singular values (sv), and setting the remaining sv's to zero. The value k is the regularization parameter which can be determined using synthetic data as a function of expected array element position accuracy, and is a function of frequency. Figure 4 illustrates the Per Christian Hansen regularization method for the annular array, where k = 43.

c) compute $\mathbf{C} = \mathbf{v} * \mathbf{si} * (\mathbf{u}')$ .

### d) Constrained Deconvolution

The second step in computing **W** is to select the smoothing kernel **B**. The **B** matrices are chosen to have desirable sidelobe levels while also having mainlobe shapes which have the same or slightly-smaller widths than the corresponding CBF mainlobe. In the case of the horizontal planar array, $\mathbf{B} = |\hat{\mathbf{A}}|^m$, where $\hat{\mathbf{A}}$ is an "average" beampattern over the set represented by **A.** Figure 3b shows the matrix **B** computed for an annular array at 400Hz, where m=3. Figure 5 compares the 23d row of |**A**| and **B** for the annular array.

### e) Final Weight Matrices

The last step in computing **W** is to form the product ( **B * C** ). Figure 6 shows the magnitude of the matrices **W** for the horizontal annular array for nine frequencies: 100, 200, 300, 400, 500, 600, 700, 800, and 900Hz.

## Sidelobe Suppression

As an example of weight-matrix performance, the two arrays are assumed to be receiving a zero-elevation, single azimuth far-field plane wave whose phase structure is given by one of the vectors in **S** . Figures 7a and 7b show decibel plots which compare the resulting beamformer output without and with the application of the deconvolutional weight matrix **W**(f), for ten frequencies spanning 100-1000Hz. As can be seen, significant reduction of both peak and average sidelobe levels is seen throughout the band.

## DBF Linearity Response

Though non-linear methods are used to compute the weight matrices W, the application of these weights is a linear operation, and the principle of superposition of multiple targets in the resulting beampattern is still applicable. Examples of this are shown in Figure 8 for three plane-wave arrivals for the horizontal annular array.

## Sensitivity

Sensitivity of the weight matrices W to error in array element position can be computed as a function of the regularizing parameter k . Figure 9 shows the result of computing the mainlobe-peak to average-sidelobe-height ratio for CBF and for DBF, for the horizontal annular planar array at 400Hz. As can be seen, the suppression of sidelobes is most marked for the case of zero positional error, but even for the "worst case" of ë/10 positional error, some sidelobe reduction versus CBF is still realized. Figure 10 shows the average sidelobe performance, relative to CBF, for the two exemplar arrays, vs. frequency from 100 to 1000Hz.

## Summary

For planar non-uniformly-spaced arrays, we have shown how to construct weight matrices with the following attributes:

1) one matrix per frequency to suppress sidelobes.
2) uses regularized, constrained deconvolution.
3) applied in beam space.
4) output beamwidths retained or improved
5) can be precomputed and stored for use (not “adaptive”).
6) achieves linear response for simultaneous arrivals
7) can be optimized vs. expected position error.

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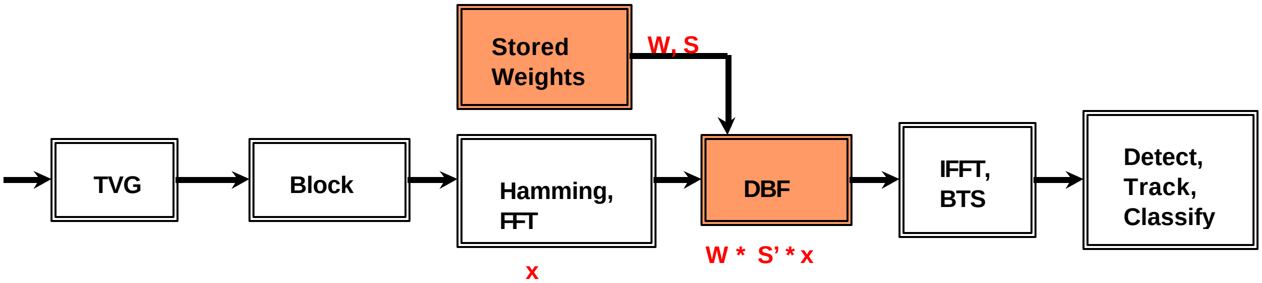



*Figure 1a. Generic processing string in which the application of the subject weight matrix* **W** *is performed*
*(TVG=time-varying gain; DBF=deconvolutional beamformer; BTS=bearing time series) .*

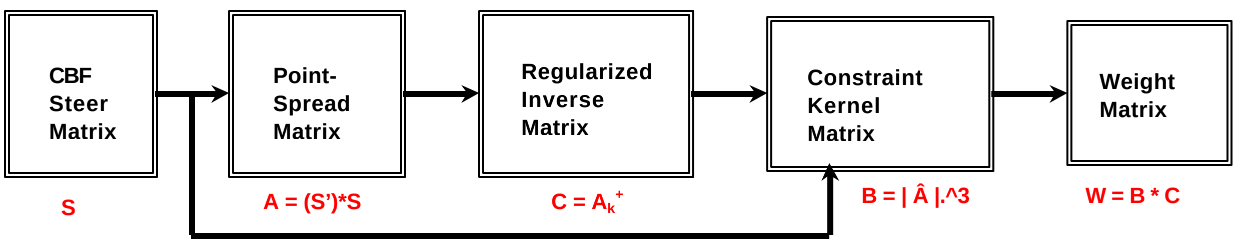
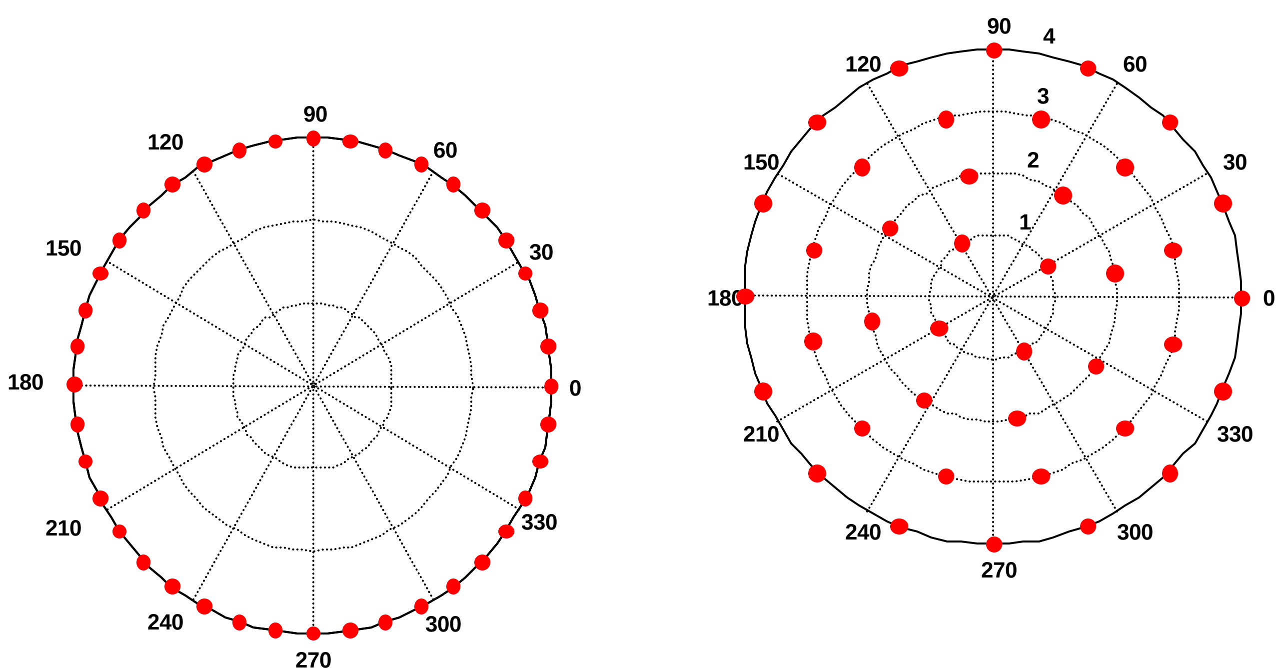



*Figure 1b. Weight matrix* **W** *algorithm flow.*




(a) (b)

*Figure 2. Exemplar Arrays, M=40: (a) horizontal planar, annular, L=60m; (b) horizontal planar, filled, L=8m.*

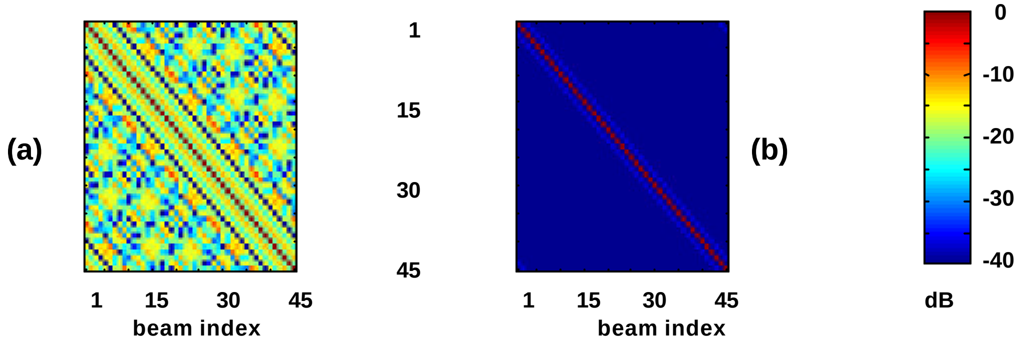



*Figure 3. Beampattern matrices for the horizontal annular planar array, at frequency 400Hz : (a) |**A**| ; (b) **B**.*

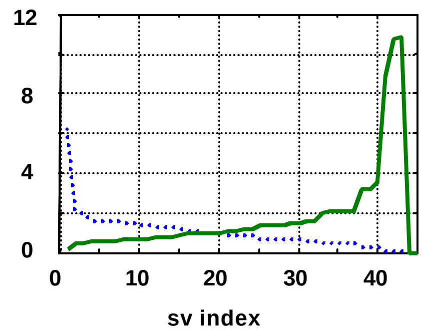



*Figure 4. Singular values (dot/blue) and inverse singular values (solid/green), for f=400Hz, for the horizontal annular planar array.*

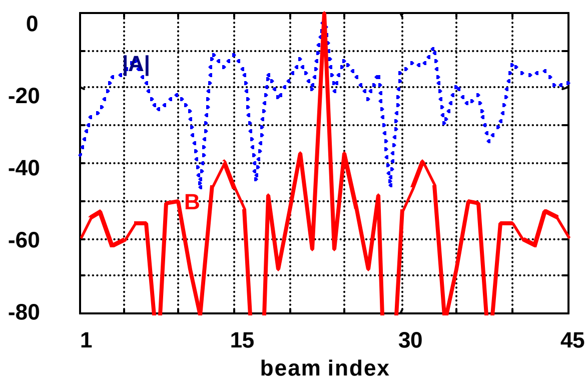



*Figure 5. Sections of beampattern matrices, row 23, horizontal planar array, 400Hz, dot/blue: |**A**|; solid/red: **B** .*

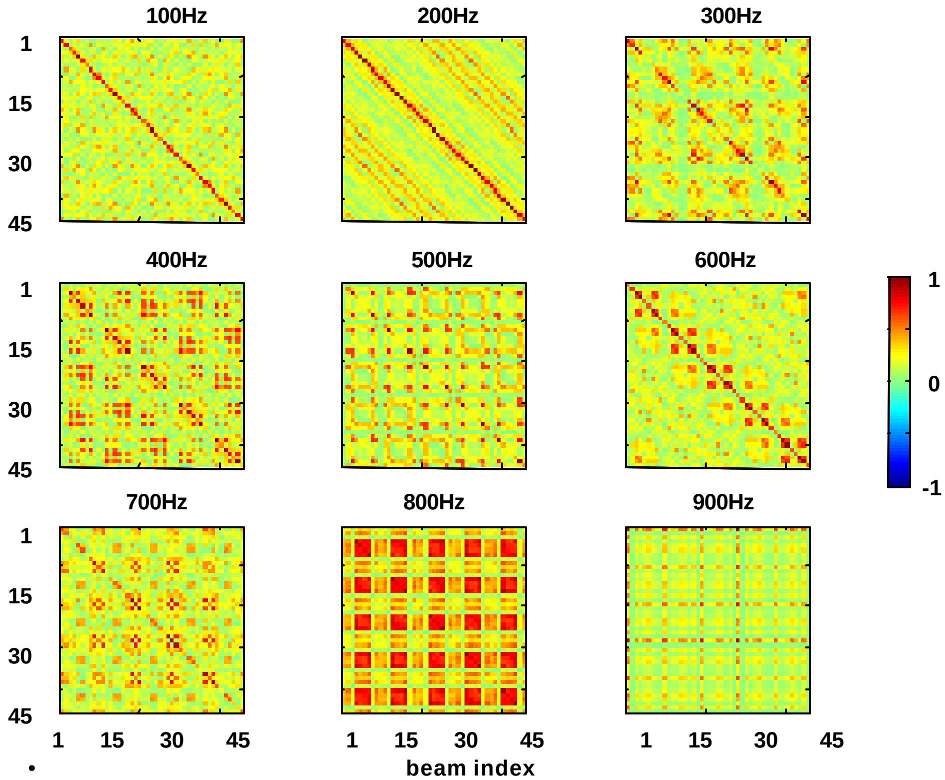
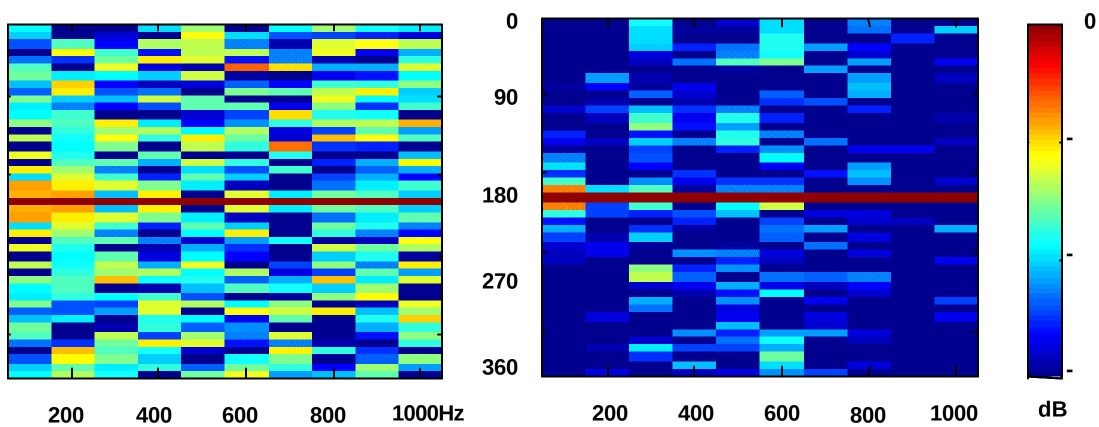
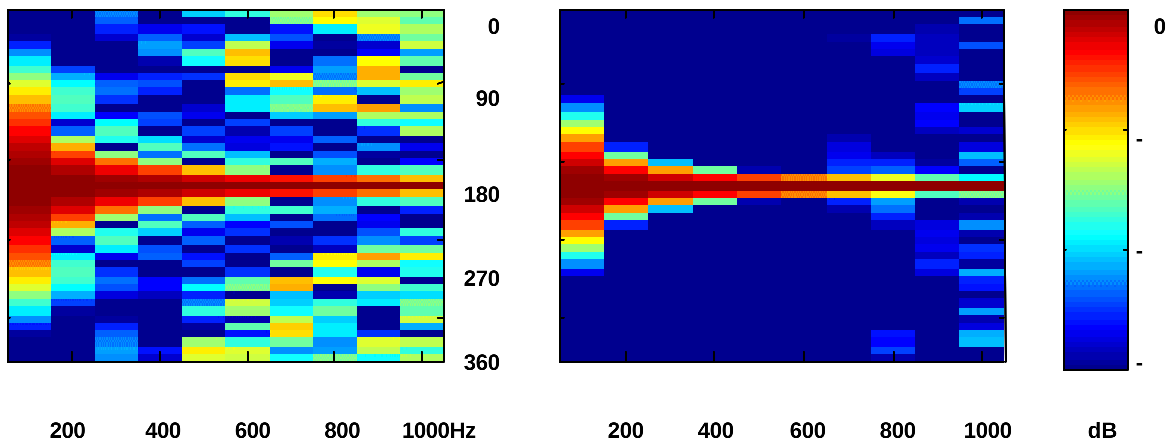



*Figure 6. The magnitude of the weight matrix* **W** *at nine frequencies: horizontal annular planar array*




(a) (b)

*Figure 7a. For the horizontal planar/circular array: comparison of beamformer outputs at ten frequencies for a plane-wave arrival at 180 deg: (a) CBF; (b) DBF .*




(a) (b)

*Figure 7b. For the horizontal planar/filled array: comparison of beamformer outputs at ten frequencies for a plane-wave arrival at 180 deg: (a) CBF; (b) DBF .*

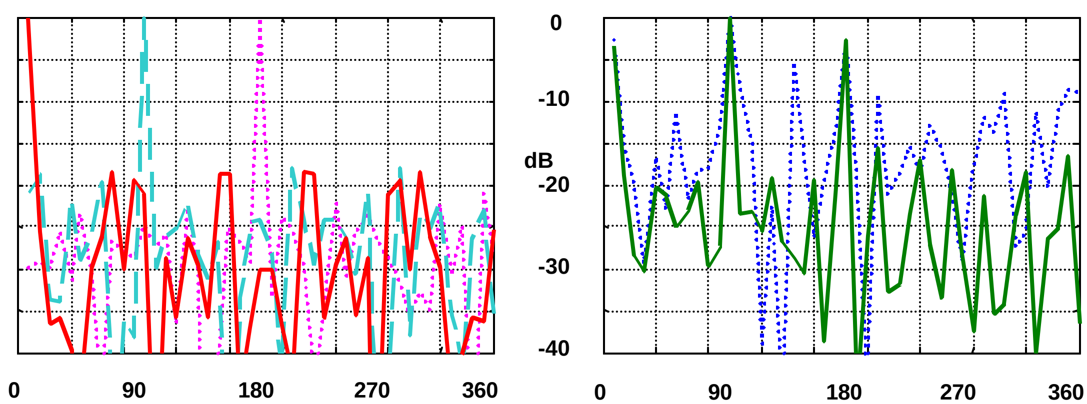



*Figure 8. DBF Linearity Response of horizontal annular planar array: (a) individual arrivals; solid/red, arrival at zero deg; dash/cyan, arrival at 90 deg; dot/magenta, arrival at 180 deg; (b) simultaneous arrivals. dot/blue, CBF; solid/green, DBF.*

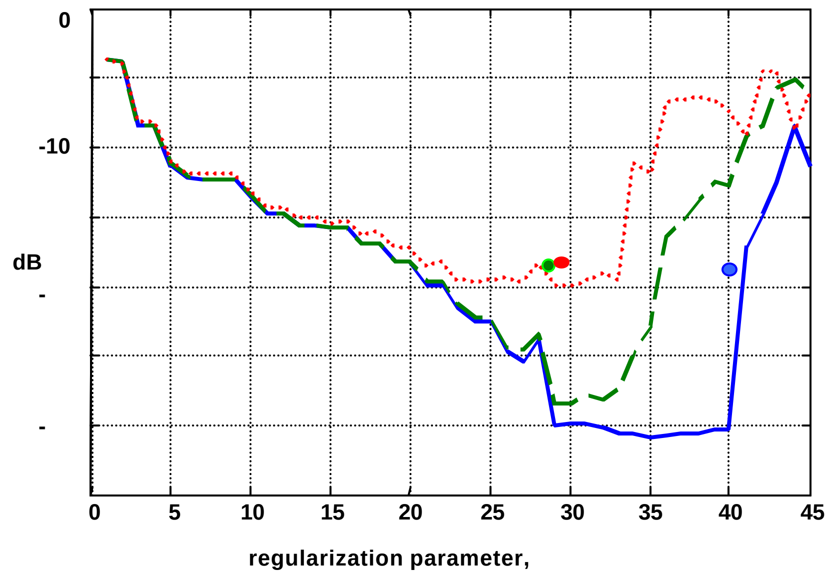



*Figure 9. Sensitivity to positional errors, horizontal annular planar array; average sidelobe level for mainlobe on the 23d beam, 400Hz; curves show DBF levels, dots show CBF levels; solid/blue: error = 0; dash/green: error = .02l; dot/red: error = .10l.*

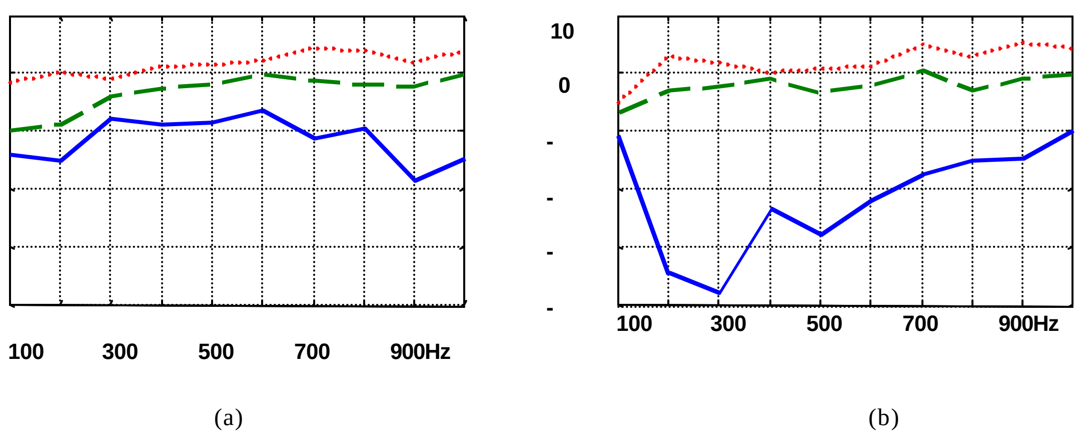



*Figure 10. Horizontal planar array, difference in average sidelobe level between CBF and DBF vs. frequency: (a) annular; (b) filled; solid/blue: error = 0; dash/green: error = .02l; dot/red: error = .10l.*