Weight Functions for Two-Dimensional Arrays using Regularized Deconvolution

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Objective

Improve Array Beamformer Performance

- for arrays with non-uniformly-spaced elements
- via application of beam-space weights

Approach

Suppress sidelobes

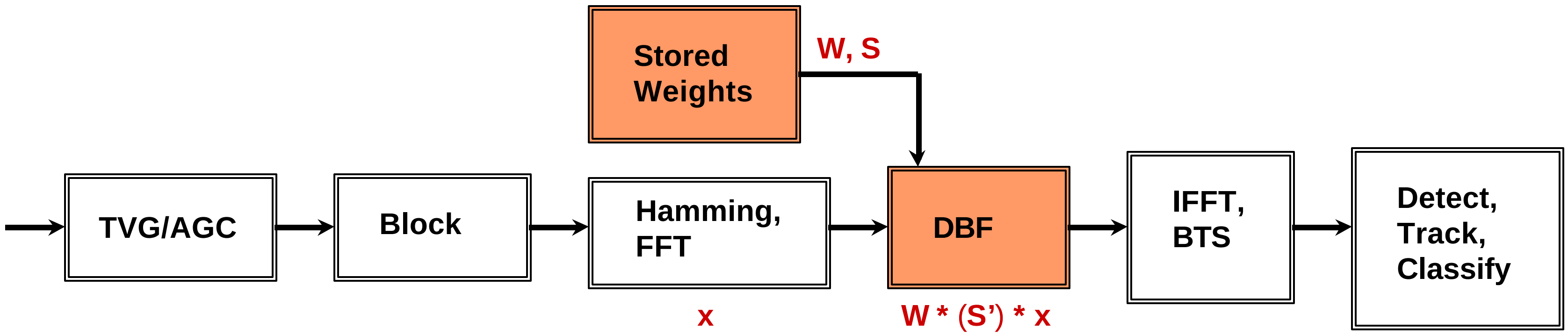
- retaining or improving mainlobe resolution
- with fixed weight matrices

Implementation

- **Formulate beamformer as a Fredholm equation**
  - **use point spread function (psf) kernel concept**
  - **kernel is look-direction dependent**

- **Use modern deconvolution methods**
  - **use pseudo-inverse of the Fredholm kernel**
    - **i.e., pseudo-inverse of the psf**
  - **regularize to achieve numerical stability**
  - **constrain to achieve robust performance**

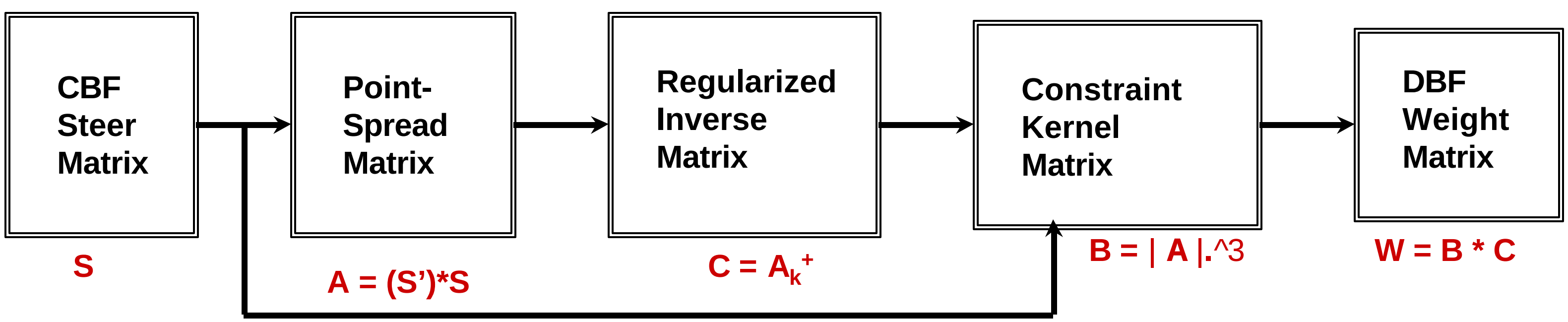
Processing Flow



TVG: time-varying gain
AGC: automatic gain control

BTS: beam time series
DBF: deconvolutional beamforming

Algorithm for Stored Weights



Algorithm Overview

1. Observed vs. “ideal” beampattern

$$\mathbf{g} = \mathbf{A} * \mathbf{f}$$

2. Observed beampattern vs. element data

$$\mathbf{g} = \mathbf{S}' * \mathbf{x}$$

3. Point-spread function vs. steering matrix

$$\mathbf{A} = \mathbf{S}' * \mathbf{S}$$

4. Solve for “ideal” beampattern

$$\mathbf{f} = (\mathbf{A}_k^{+}) * \mathbf{S}' * \mathbf{x}$$

5. Desired vs. “ideal” beampattern

$$\mathbf{h} = \mathbf{B} * \mathbf{f}$$

6. Desired beampattern vs. weight matrix

$$\mathbf{h} = \mathbf{W} * \mathbf{S}' * \mathbf{x}$$

7. Reformulate for signal phase property

$$\mathbf{h} = | \mathbf{W} * \mathbf{S}' * \mathbf{x} | .* \exp(j * \text{angle}(\mathbf{g}))$$

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Algorithm - 1

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1. $g = A * f$

f = "ideal" vector of target azimuths

- spanning 0-360 degrees
- e.g., 0:8:352, 45x1

g = observed beampattern vector

- spanning 0-360 degrees, e.g., 45x1
- includes mainlobe & sidelobes
  - superposed for each "target"

A = point-spread matrix

- delta-function in, beampattern out
- e.g., 45x45

2. $g = S' * x$

g = observed beampattern vector

- spanning 0-360 degrees, e.g., 45x1
- includes mainlobe & sidelobes

x = observed element data vector

- at single frequency
- complex; e.g., 40x1

S = steer matrix

- one column per look direction
- phase-only; e.g., 40x45

3. $A = S' * S$

S = steer matrix

- one column per look direction
- phase-only; e.g., 40x45

A = point-spread matrix

- one CBF beampattern per row
- single frequency; e.g., 45x45

4. Combine equations:

$$A * f = S' * x$$

Solve for f :

$$f = (A_k^+) * S' * x$$

A_k^+ = regularized inverse of A

using first k singular values

Algorithm - 2

5. $\mathbf{h} = \mathbf{B} * \mathbf{f}$

f = "ideal" vector of target azimuths

h = desired beampattern vector response

- mainlobe not too narrow
- sidelobes reduced from g

B = smoothing constraint matrix

- delta-function in, improved beampattern out
- e.g., 45x45

6. Combine equations:

$$\mathbf{h} = \mathbf{B} * (\mathbf{A}_k^{+}) * \mathbf{S}' * \mathbf{x}$$

rewrite :

$$\mathbf{h} = \mathbf{W} * \mathbf{S}' * \mathbf{x}$$

where

$$\mathbf{W} = \mathbf{B} * (\mathbf{A}_k^{+})$$

W represents stored weight matrix

- beamspace matrix
- single frequency, NxN

7. Reformulate for signal phase property :

$$\mathbf{h} = |\mathbf{h}| .* \exp(j * \text{angle}(\mathbf{g}))$$

then :

$$|\mathbf{h}| = | \mathbf{W} * \mathbf{S}' * \mathbf{x} |$$

where

$$\mathbf{W} = \mathbf{B} * (\mathbf{A}_k^{+})$$

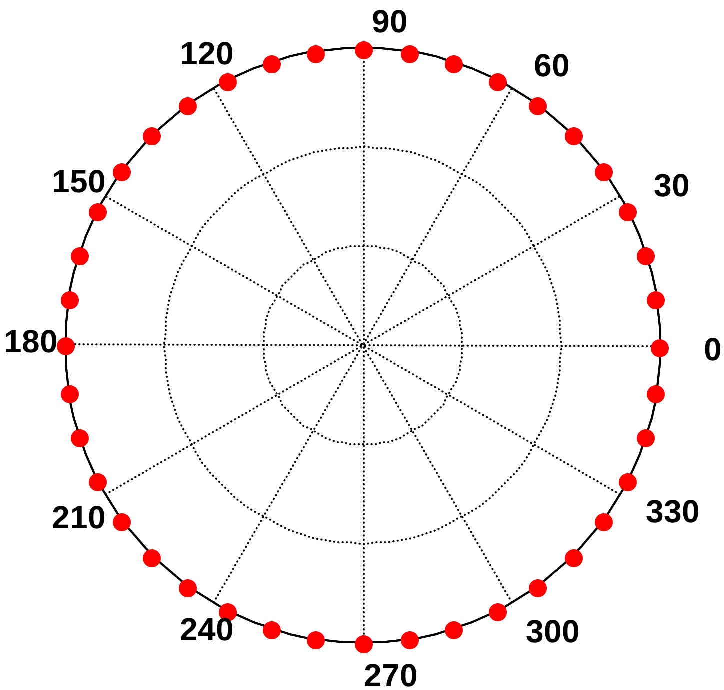
(don't want to mess with original phase of g, anyway)

Details/examples below

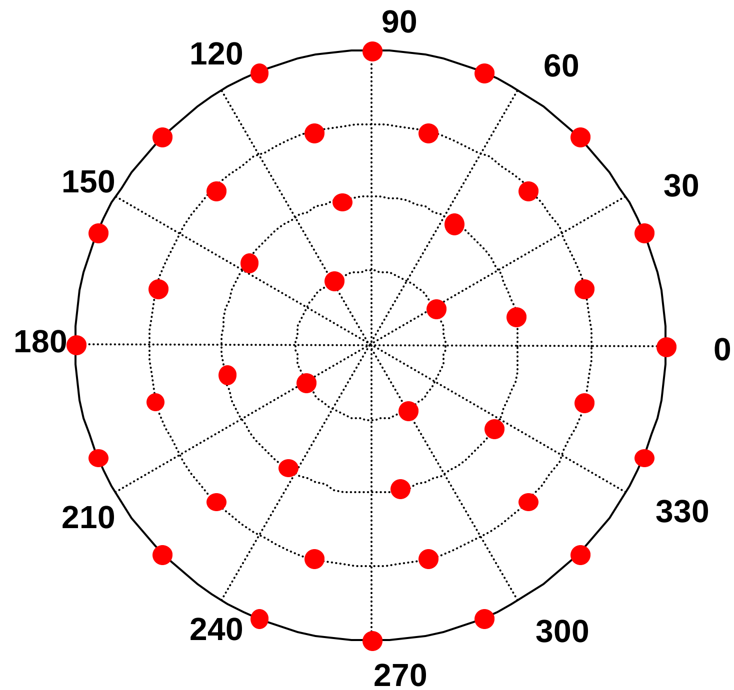
- how to get (A_k^{+})
- how to select B
- appearance vs. frequency
- sensitivity to errors

Array Examples

L M I



Horizontal Planar Array, Annular
D = 60m, M = 40



Horizontal Planar Array, Filled
D = 8m, M = 40

A, B Matrices, 400Hz

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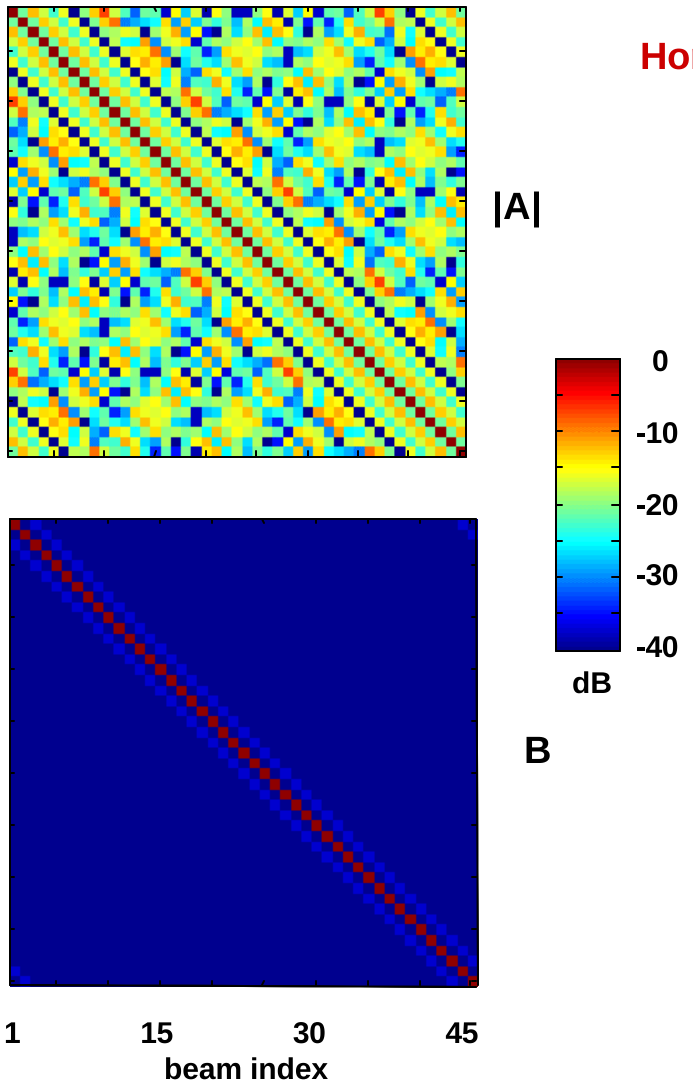
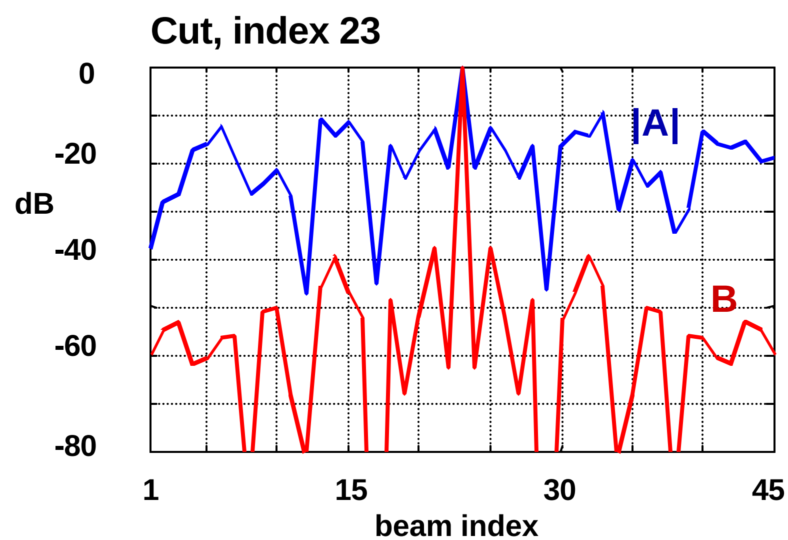
Horizontal Planar Array, Annular





Computation Details

L M I

(1) [u,s,v] = svd(A)

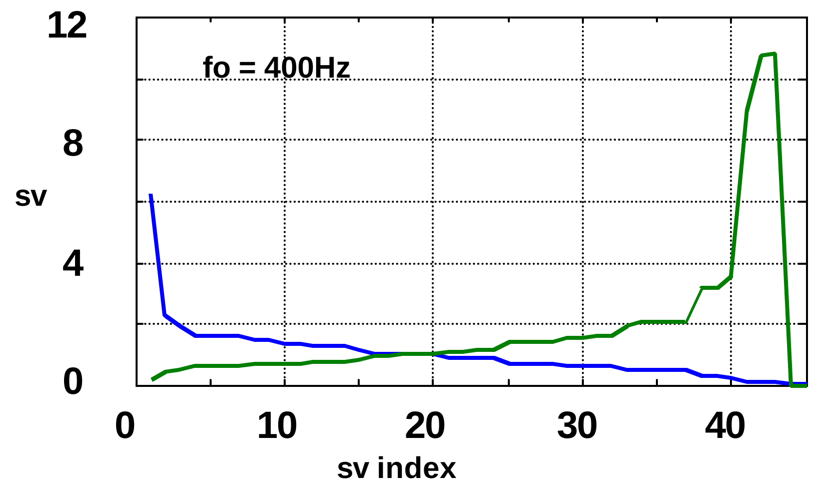
(2) si = inverse of s via Per Christian Hansen method
(keep k singular values, invert each, make remaining ones zero)

(3) A_k^+ = v * si * (u') ---- regularized inverse

(4) B = | A | .^ (3) ---- constraint/smoothing matrix

(5) W = B * (A_k^+)

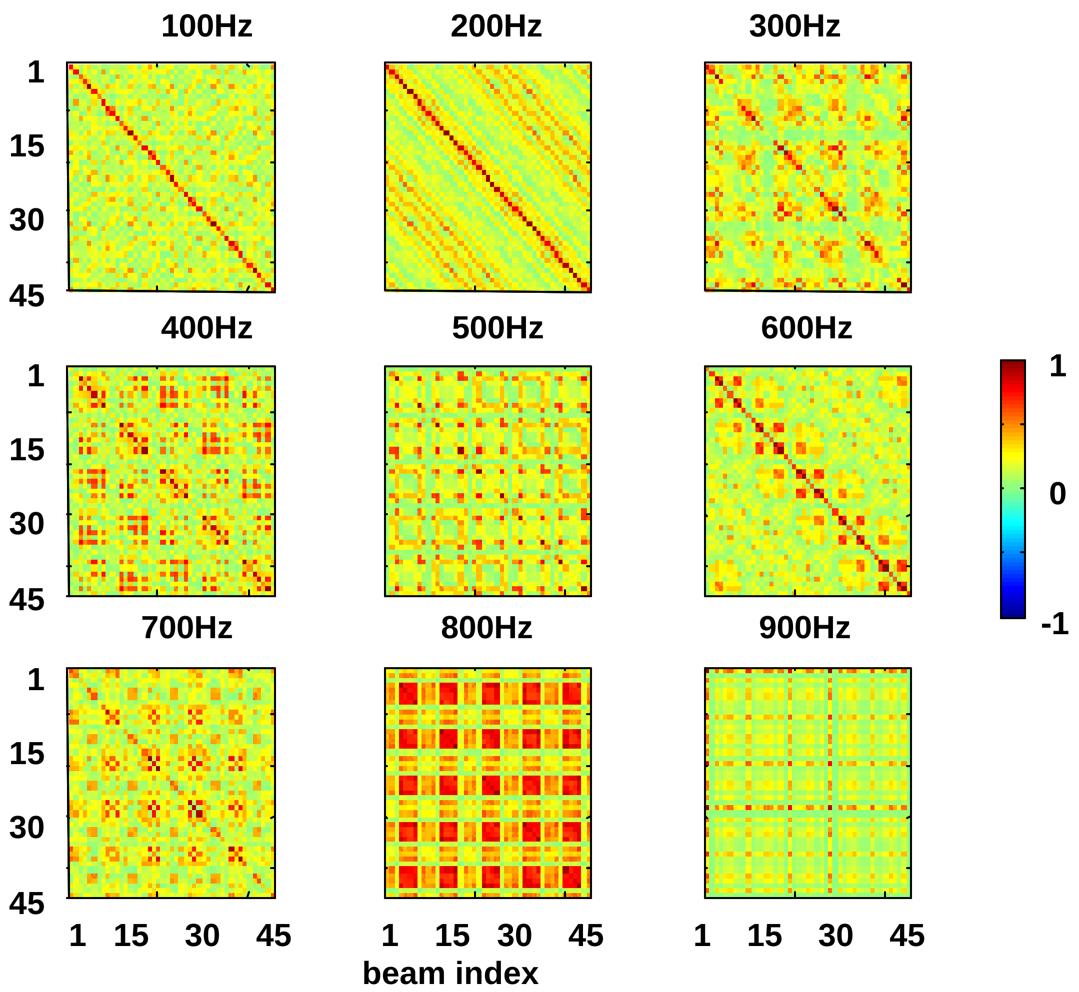
Horizontal Planar Array, Annular



A = average point-spread matrix
sv = singular value

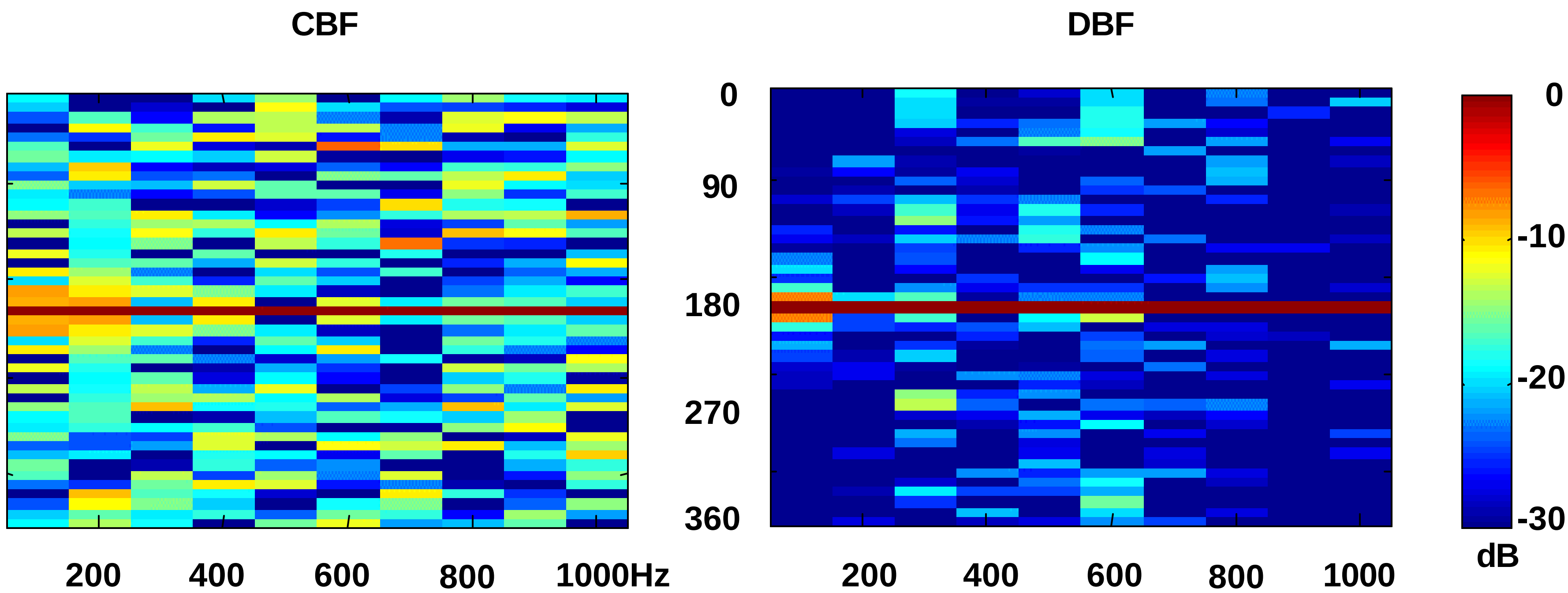
Weight Matrix W

Horizontal Planar Array, Annular



CBF & DBF vs. Frequency

Horizontal Planar Array, Annular

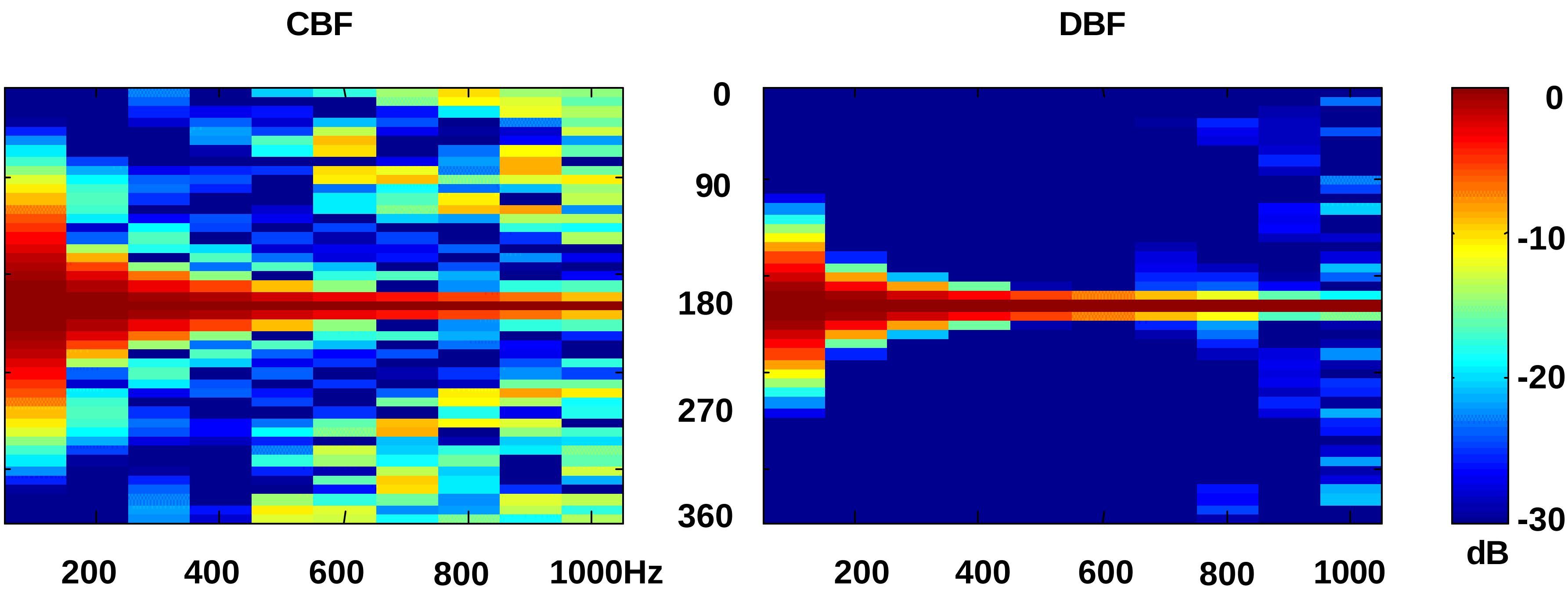


CBF = conventional beamformer

DBF = deconvolutional beamformer

CBF & DBF vs. Frequency

Horizontal Planar Array, Filled



CBF = conventional beamformer

DBF = deconvolutional beamformer

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DBF Linearity Response

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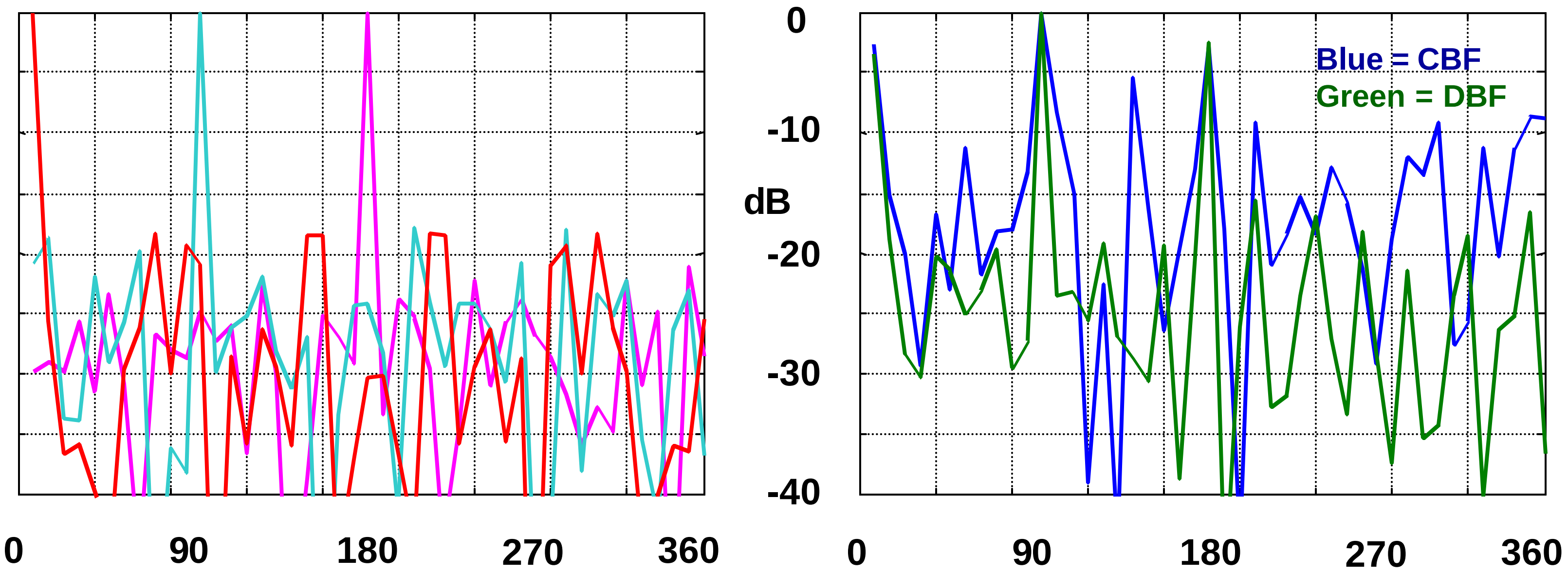
Horizontal Planar Array, Annular

Individual arrivals

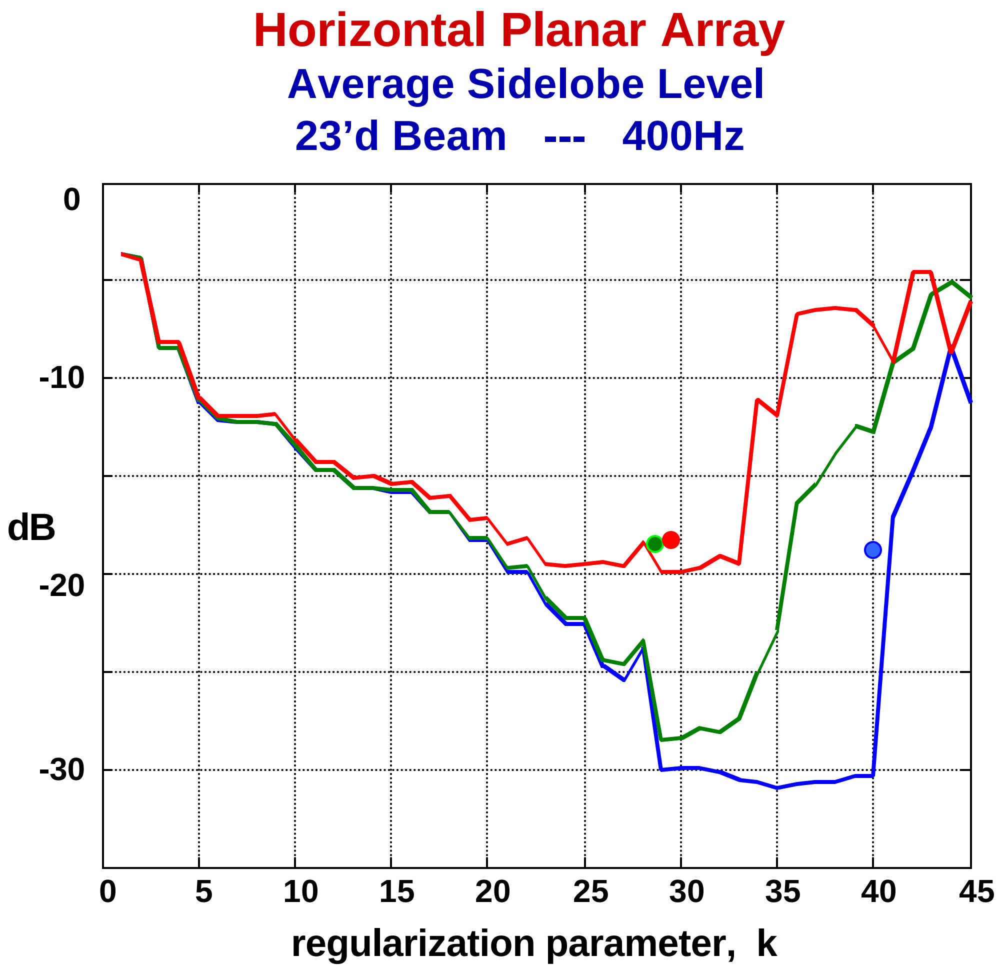
Simultaneous arrivals



CBF = conventional beamformer

DBF = deconvolutional beamformer

Sensitivity to Position Error



Error in lambda
CURVES: DBF
BLUE:
error = 0
GREEN:
error = .02
RED:
error = .10
DOTS: CBF

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Sensitivity to Position Error

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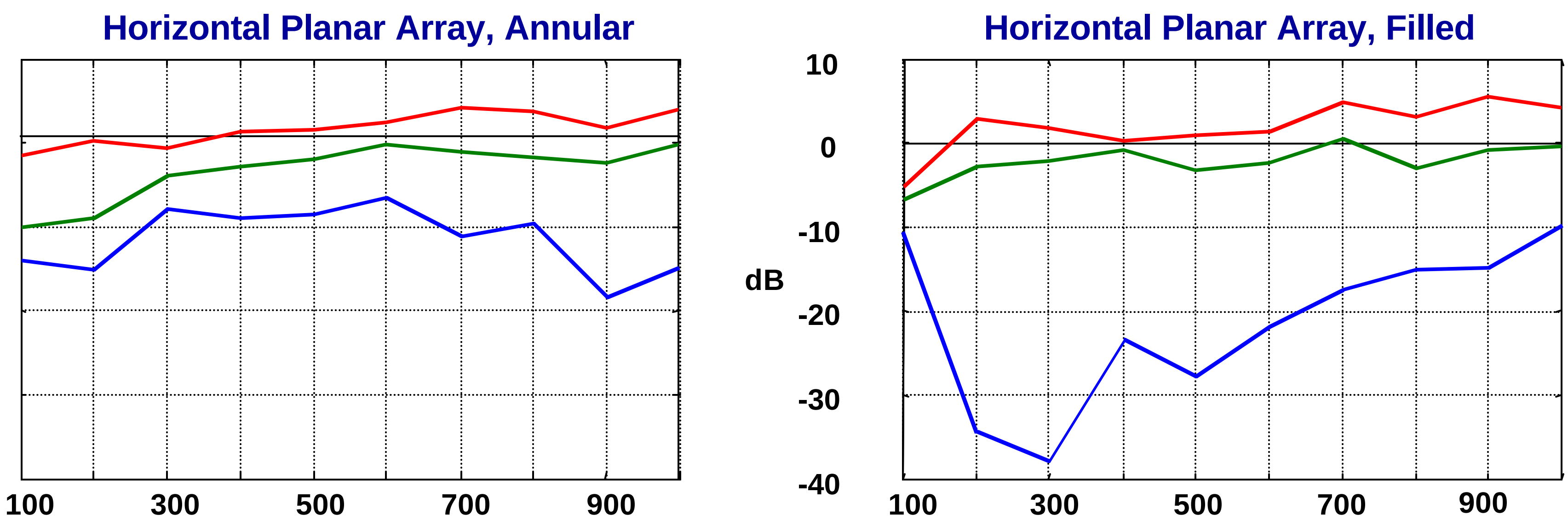
Average Sidelobe Level Difference

DBF relative to CBF, at 10 frequencies across band



BLUE: error = 0
GREEN: error = .02 λ
RED: error = .10 λ

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Summary

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Have shown how to construct weight matrices

- for planar, non-uniformly-spaced arrays
- one matrix per frequency to suppress sidelobes
- using regularized, constrained deconvolution
- applied in beam space
- output beamwidth retained or improved
- precomputed, stored for use (not “adaptive”)
- complementary to (compatible with) ABF
- achieve linear response for simultaneous arrivals
- can optimize vs. expected position error

ABF = adaptive beamformer

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References

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B A C K U P

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Background

LMI

- Program: Nonlinear Dynamical Stochastic Resonance
- Task: Non-Linear Aperture Processing
- Sponsor: NAVAIR, PMA-264, Capt. Randy Loyer
- Manager: SSC/SD, Dr. David Rees, D857
- Task Team Members:
  - James Alsup AETC
  - Harper Whitehouse LMI
  - Shelby Sullivan Sr. SAIC

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Algorithm - 1

$$g = A * f$$

f = "ideal" vector of target azimuths
- spanning 0-360 degrees
- e.g., 0:8:352, 45x1

g = observed beampattern vector
- spanning 0-360 degrees, e.g., 45x1
- includes mainlobe & sidelobes
  - superposed for each "target"

A = point-spread function
- delta-function in, beampattern out
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Algorithm - 2

$$g = S' * x$$

g = observed beampattern vector
- spanning 0-360 degrees, e.g., 45x1
- includes mainlobe & sidelobes

x = observed element data vector
- at single frequency
- complex; e.g., 40x1

S = steer matrix
- one column per look direction
- phase-only; e.g., 40x45

Algorithm - 3

$$A = S' * S$$

S = steer matrix
- one column per look direction
- phase-only; e.g., 40x45

A = point-spread matrix
- one CBF beampattern per row
- single frequency; e.g., 45x45

Algorithm - 4

Combine equations:

$$A * f = S' * x$$

Solve for f :

$$f = (A_k^+) * S' * x$$

A_k^+ = **regularized inverse of** A

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Algorithm - 5

LMI

$$h = B * f$$

f = "ideal" vector of target azimuths

h = desired beampattern vector response

- mainlobe not too narrow
- sidelobes reduced from g

B = smoothing constraint matrix

- delta-function in, improved beampattern out
- e.g., 45x45

Algorithm - 6

Combine equations:

$$h = B * (A_k^+) * S' * x$$

Rewrite :

$$h = W * S' * x$$

where

$$W = B * (A_k^+)$$

W represents stored weight matrix

- **beamspace matrix**
- **single frequency, NxN**

Algorithm - 7

Reformulate for signal phase property :

$$\mathbf{h} = |\mathbf{h}| \; .* \; \exp(j * \text{angle}(\mathbf{g}))$$

Then :

$$|\mathbf{h}| = | \mathbf{W} * \mathbf{S'} * \mathbf{x} |$$

where

$$\mathbf{W} = \mathbf{B} * (\mathbf{A_k^+})$$

(don't want to mess with original phase of g, anyway)